Roots of Polynomial Equations 1 - MS

Q1.	Uses substitution $y = x^3$	M1
	Obtains $y + y^{1/3} - 1 = 0$	A1
	$y = (1 - y)^3$	A1
	$\Rightarrow \ldots \Rightarrow y^3 - 3y^2 + 4y - 1 = 0$ (AG)	A1
	$\sum \alpha^6 = \left(\sum \alpha^3\right)^2 - 2\sum \beta^3\gamma^3$	B1
	$= 9 - 8 = 1$	M1A1
	OR put $y = z^{1/2}$ to obtain	
	$z^3 - z^2 + 10z - 1 = 0$	M1A1
	$\sum \alpha^6 = $ –coefficient of z^2, $= 1$	A1

Q2.	**(i)**	α a root of given equation $\Rightarrow \alpha^4 - 5\alpha^2 + 2\alpha - 1 = 0$	
		$\Rightarrow \alpha^{n+4} - 5\alpha^{n+2} + 2\alpha^{n+1} - \alpha^n = 0$	M1
		Summing over α, β, γ, δ, leads to $S_{n+4} - 5S_{n+2} + 2S_{n+1} - S_n = 0$	A1
	(ii)	$S_2 = 10$	B1
		$S_4 = 5S_2 - 2S_1 + 4 = 50 - 0 + 4 = 54$	M1A1
	(iii)	$S_{-1} = 2$ from e.g., $y^4 - 2y^3 + 5y^2 - 1 = 0$	M1A1
		$S_3 = 5S_1 - 2S_0 + S_{-1} = -6$	M1A1
		OR	
		$2S_3 = 3S_1S_2 - S_1^3 + 6\sum\alpha\beta\gamma$	M1A1
		$= 3 \times 10 \times 0 - 0 + 6 \times (-2)$	M1
		$\Rightarrow S_3 = -6$	A1
		$S_6 = 5S_4 - 2S_3 + S_2 = 292$	M1A1
	(iv)	$\sum \alpha^2\beta^4 = S_2S_4 - S_6 = 540 - 292 = 248$	M2A1

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Q3.	$x = y^{1/3} \Rightarrow y^4 = (1+y)^3$	M1A1
	$\Rightarrow y^4 - y^3 - 3y^2 - 3y - 1 = 0$	A1
	$\sum \alpha^6 = 1 - 2\times(-3) = 7$	M1A1
	or $y = x^2 \Rightarrow y^4 - y^3 - 2y^2 + 1 = 0$ has roots $\alpha^2,\ \beta^2,\ \gamma^2$	M1
	$\sum \alpha^6 = \left(\sum \alpha^2\right)\left(\sum \alpha^4\right) - \left(\sum \alpha^2\right)\left(\sum \alpha^2\beta^2\right) + 3\left(\sum \alpha^2\beta^2\gamma^2\right)$	M1A2
	$= 5 - (-2) + 0 = 7$	A1
	or For last 2 marks	
	$z = y^2 \Rightarrow z^4 - 7z^3 + z^2 - 3z + 1 = 0$	M1
	$\sum \alpha^6 = -\frac{(-7)}{1} = 7$	A1
	or Use of $S_{N+4} = S_N + S_{N+3}$	M1
	$S_{-1} = \frac{0}{1} = 0$ $\quad S_2 = 1^2 - 2\times 0 = 1$	(both) M1A1
	$S_3 = 0 + 1 = 1$ $S_4 = 1 + 4 = 5$	(both) A1
	$S_5 = 5 + 1 = 6$ $S_6 = 6 + 1 = 7$	(both) A1

Q4.	Shows $y = -3/x \Rightarrow y^3 - 5y^2 - 9 = 0$	B1
	$y = -3/x \Rightarrow y = \alpha\beta\gamma/x$	M1
	$\Rightarrow y = \beta\gamma, \gamma\alpha, \alpha\beta$ when $x = \alpha, \beta, \gamma$, respectively	M1A1
	OR for previous 3 marks:	
	$\beta\gamma + \gamma\alpha + \alpha\beta = 5$	B1
	$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \alpha\beta\gamma(\alpha + \beta + \gamma) = \alpha\beta\gamma \times 0 = 0$	B1
	$\beta\gamma\gamma\alpha\alpha\beta = (\alpha\beta\gamma)^2 = 9$	B1
	$\Sigma\alpha^2\beta^2 = 25 - (2\times 0) = 25$	M1A1
	$\Sigma\alpha^3\beta^3 - 5\,\Sigma\alpha^2\beta^2 - 27 = 0$	M1A1
	$\Rightarrow \ldots \Rightarrow \Sigma\alpha^3\beta^3 = 152$	A1

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Q5.	Shows $y = -3/x \Rightarrow y^3 - 5y^2 - 9 = 0$	B1
	$y = -3/x \Rightarrow y = \alpha\beta\gamma/x$	M1
	$\Rightarrow y = \beta\gamma, \gamma\alpha, \alpha\beta$ when $x = \alpha, \beta, \gamma$, respectively	M1A1
	OR for previous 3 marks:	
	$\beta\gamma + \gamma\alpha + \alpha\beta = 5$	B1
	$\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2 = \alpha\beta\gamma(\alpha + \beta + \gamma) = \alpha\beta\gamma \times 0 = 0$	B1
	$\beta\gamma\gamma\alpha\alpha\beta = (\alpha\beta\gamma)^2 = 9$	B1
	$\Sigma\alpha^2\beta^2 = 25 - (2 \times 0) = 25$	M1A1
	$\Sigma\alpha^3\beta^3 - 5\,\Sigma\alpha^2\beta^2 - 27 = 0$	M1A1
	$\Rightarrow \ldots \Rightarrow \Sigma\alpha^3\beta^3 = 152$	A1

Q6.	Obtains an equation in y not involving radicals, e.g.,	
	$y(y + 1)^2 = 1$	M1
	$\Rightarrow \ldots \Rightarrow y^3 + 2y^2 + y - 1 = 0$ (AG)	A1
		[2]
	(i) $S_2 = -2$	B1
	$S_4 = 4 - 2 = 2$	M1A1
		[3]
	(ii) $S_6 = -2S_4 - S_2 + 3 = 1$	M1A1
	OR	
	$\Sigma\alpha^2 = -2,\ \Sigma\alpha^2\beta^2 = 1,\ \alpha^2\beta\gamma^2 = 1$	
	$S_6 = (\Sigma\alpha^2)^3 - 3\Sigma\alpha^2\Sigma\alpha^2\beta^2 + 3\alpha^2\beta^2\gamma^2$	M1
	$= (-2)^3 - 3 \times (-2) \times 1 + 3$	
	$= -8 + 6 + 3$	
	$= 1$	A1
	$S_8 = -2S_6 - S_4 + S_2 = -6$	M1A1
		[4]

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Q7. **(i)** $x = 1/y \Rightarrow 2y^4 - 4y^3 - cy^2 - y - 1 = 0$ M1A1

[2]

(ii) $\sum \alpha^2 = 1 - 2c$ M1A1

$\sum \alpha^{-2} = 4 + c$ A1

(M1 is for use of $\sum \alpha^2 = \left(\sum \alpha\right)^2 - 2\sum \alpha\beta$ in either part.) [3]

(iii) $S = \sum \left(\alpha - \alpha^{-1}\right)^2 = \sum \alpha^2 + \sum \alpha^{-2} - 8 = -c - 3$ M1A1√

A1ft is for adding answers to **(ii)** correctly and subtracting 8. [2]

(iv) $c = -3 \Rightarrow S = 0$ so that if all roots are real then $\alpha = \pm 1$

and similarly for β, γ, δ M1A1 CWO

This is impossible since e.g., $\alpha\beta\gamma\delta = -2$, or any other contradiction A1 CWO

[3]